Stability Analysis of Laminated Composite Twisted Plates with Holes

A Thesis Submitted in Partial Fulfillment of the Requirements for the degree of

Master of Technology
In
Civil Engineering

Varre Ananth Naga Kumar

ROLL NO: 710CE2010

(Department of Civil Engineering)

Under the guidance of

Prof. A.V. Asha

Department Of Civil Engineering

National Institute Of Technology, Rourkela

CERTIFICATE

This is to certify that the thesis entitled "**Stability Analysis of Laminated Composite Twisted Plates with Holes**" submitted by **Mr. Varre Ananth Naga Kumar** in partial fulfillment of the requirement for the award of **Master of Technology** Degree in **Civil Engineering** at **National Institute of Technology Rourkela** is an authentic work carried out by him under my guidance and supervision.

To the best of my knowledge, the matter presented in this thesis has not been presented in any other university/college for any other degree or diploma.

Date: Prof. A.V. Asha

Place: Rourkela Department of Civil Engineering

Rourkela

ACKNOWLEDGEMENTS

I would like to express my gratitude to my guide **Prof.A.V.Asha** for her constant advice, support for the betterment of my research work and proper mentoring throughout my studies. Her technical advice has played a key role in the completion of the thesis in the stipulated time. Her teaching ability, depth of knowledge and a perfect insight into the problems will always be an inspiration to me and all her students.

I thank also **Prof. S. K. Sarangi**, **Director, NIT Rourkela** and **Prof. S.K.Sahu**, **Head of the Civil Engineering Department, NIT Rourkela**, for providing me with the necessary facilities to carry out my research.

I am very thankful to all the faculty members and staffs of civil engineering department who assisted me in my research, as well as in my undergraduate studies.

I also thank my batch mates who directly or indirectly have helped me a lot to complete my research work. I also thank my M.Tech Structural student friends Indrajeet and Bala Murali Krishna for their valuable advice for my project.

Varre Ananth Naga Kumar
Roll No : 710CE2010
Department of Civil Engineering
National Institute of Technology, Rourkela

ABSTRACT

A composite material is formed by the combination of two or more individual components of materials which have different unique properties. Thus this combination leads to a material which has better properties than the materials from which it is formed. This paper aims to evaluate the buckling load of square cross ply laminated composite twisted plate with circular cutouts with the variation of different parameters. The different types of parameters considered in this study are: (a) cutout location (b) cutout size (c) different layups (d) angle of twist and (e) aspect ratio. Uniaxial compressional loading is considered throughout the paper. Due to the presence of cutouts in the plates, the stress concentration is near to the holes and the stiffness of the plates is reduced. The analysis considering the above factors is made using ANSYS software. An eight node element is considered in the present analysis with six degrees of freedom per node. The SHELL 281 with six degrees of freedom per node is used. The effect of angle of twist, number of layers, hole diameter, location of hole etc on buckling loads are presented in this paper.

It is found that with the increase of the angle of twist of laminated composite twisted plate with different holes, the buckling load decreases. Also as the hole diameter increases the buckling load decreases for different ply layups. Different types of models are created and buckling analysis is carried out considering various parameters.

Keywords : laminated composite plate, cutouts, buckling, twisted plate

Contents

List of tables

List of figures

CHAPTER 1

1.1 INTRODUCTION

A composite material is formed by the combination of two or more individual components of materials which have different unique properties. Thus this combination leads to a material which has better properties than the materials from which it is formed. This material does not blend or dissolve into one another and the different materials within the composite can be easily distinguished. The materials formed are lighter, stronger and less expensive when compared to other materials. There are various types of engineered composite materials such as composite building material which include concrete, cement and ceramic composites, metal composites and laminated composites. The popularity of composite materials have made many researchers investigate their behaviour and obtain a better understanding of them. There are many advantages of composite materials over other materials such as concrete, steel, etc as they have high strength to weight ratio and stiffness to weight ratios. This high strength and stiffness ratio is the reason for their increasing use in aerospace industry and automative works. Buckling is one of the main failure mechanisms in composite plates. Accurate prediction of structural response characteristics is a challenging problem in the analysis of laminated composites due to the orthotropic structural behaviour, the presence of various types of couplings and due to less thickness of the structural elements made of composites. Thus, an accurate buckling analysis of the laminated composite plates is an important part of the structural design. Hence the buckling behaviour of these kinds of plates should be analysed properly. Buckling behaviour of laminated composite plates subjected to in-plane loads is an important aspect in the preliminary design of aircraft and launch vehicle components. Holes are provided either in the center or elsewhere in the laminar plates for the purpose of pipes for electric cables or other purposes. Due to the presence of holes in the plates, the stress concentration is near to the holes and the stiffness of the plates is reduced. Hence the study is important in order to know the buckling behaviour of such plates.

CHAPTER 2

2.1 LITERATURE REVIEW

In the literature, there are a range of published studies on the buckling of composite plates.

Vandenbrink and Kamat (1987) researched the buckling and post-buckling behaviour of isotropic and composite plates. The plates had central holes. The post-buckling strength of composite plates showed more loss compared to isotropic plates. They also found that when the fiber orientation angle is 60 degrees, the buckling load of composite plates which have large holes was more than the corresponding solid plate.

Larsson (1987, 1989) investigated the buckling and post buckling behaviour of orthotropic laminated square and circular plates. The plate had a central circular cutout. He utilized the finite element method to solve the problem for biaxial and uniaxial compression for different boundary conditions. He concluded that the buckling load of square plates with cutouts at the center in case of uniaxial compression was reduced.

Hyer and Lee (1991) investigated the use of curvilinear fiber format to improve the buckling load of square composite plates with central holes. The study stated that the fiber orientation near the hole has more effect on the buckling load than that of the fiber orientation far away from the hole.

Hu and Lin (1995) analysed symmetrically laminated plates with central holes and presented an optimization scheme for the buckling load with respect to fiber orientation. They stated that except for simply supported thin square composite plates, the optimal buckling load increases with increase in sizes of the cutouts. Therefore it was possible to increase the buckling load with a suitable cutout size and fibre orientation angle as compared to those of plates without cutouts.

Shakerley and Brown (1996) investigated elastic buckling of simply supported and fully clamped plates using conjugate method. They placed rectangular cutouts eccentrically. The load applied was uniaxial compression and shear loading.

Akbulut and Sayman (2001) used the first order shear deformation theory to carry out a buckling analysis of a rectangular composite laminate with a central square hole. The critical buckling loads of composite plates under the in-plane loads were found for constant or varying thicknesses, various hole sizes, various modular ratios, simple loading and biaxial loading.

El-Sawy and Nazmy (2001) analysed simply supported rectangular perforated plates. The loading was uniaxial edge compression in the longitudinal direction. They used the finite element method to determine the buckling load.

Saha *et al*. (2004) analysed buckling behaviour of composite plates with central holes under compression loading. They observed that as the hole diameter increases, the effect of plate slenderness ratio on the buckling load was less.

The critical buckling load and post-buckling behaviour of laminated plates with circular and elliptical cutouts under axial compression was studied by both Jain and Kumar (2004) and Ghannadpour *et al*. (2006). Their study led to the conclusion that buckling load decreases as the central circular hole diameter increases. For a laminate without a cutout, failure occurred near the diagonal corner of the plate, but in the presence of a circular cutout, the failure location shifted towards the cutout edge. They also observed that failure took place near the vertex of the cutout if it was elliptical in shape.

Baltaci *et al*. (2006) analyzed laminated circular plates with circular holes using finite element analysis. Their conclusion was that the critical buckling load decreased as the hole got closer to the plate center.

The effect of support conditions on the buckling load of laminated plates with circular and semicircular cutouts under axial compression was studied by Baba (2007) and Baba and Baltaci (2007). They found that the buckling load of clamped-clamped plates was higher than the buckling load of simply supported plates and clamped-pinned plates. They also found that an increase of the length/thickness ratio increased the buckling load.

Elastic buckling of an isotropic rectangular plate with circular cutout under linearly varying in-plane uniaxial loading was studied by Komur and Sonmez (2008) using finite element simulation. To evaluate the effect of cutout location on the buckling load of plates, circular cutouts were chosen at different locations along the principal x-axis of the plate.

Maiorana *et al.* (2009) analysed linear buckling behaviour of square and rectangular plates subjected to axial compression and bending moment. The circular and rectangular cutouts were placed at various positions. The aim of their analysis was to suggest the best position and orientation of cutouts in steel plates, when axial compression and bending moment were applied together..

Buckling of laminated composite plates with elliptical or circular cutout was presented by Komur *et al* (2010). In this study, which was done using finite element method, the buckling of a woven–glass–polyester laminated composite plate with a circular or elliptical hole was carried out numerically. In the analysis, parametric studies on different plates taking into account the shape and position of the elliptical hole was done.

Gaira *et al.* (2012) worked out the buckling load factors for laminated composites with different aspect ratio, d/b ratio and d/D ratio .They stated that the buckling load factors were lowered due to presence of cutouts. With the increase of aspect ratio, the buckling load factor increased. The load was inversely proportional to d/D ratio up to 0.24 and also inversely proportional to d/b ratio up to 0.15.

2.2 OBJECTIVE OF THE PRESENT WORK:

A study of the literature shows considerable work has been done on the buckling of laminated composite plates. The present study mainly aims to analyse the buckling behaviour of laminated composite twisted plates with holes. The present study aims to understand buckling behaviour of laminated composite twisted plates with holes.

2.3 OUTLINE OF THE PRESENT WORK:

The present study mainly aims to deal with the behaviour of laminated composite twisted plate with different sizes of cutouts by the influence of uniaxial compressive loading. The influence of different parameters such as number of layers, angle of twist, size to thickness ratio, cutout size, cutout location and aspect ratio are examined in this study.

This thesis consists of five chapters and in the very first chapter; a small introduction of this present study has been outlined.

While coming to the chapter 2, the detailed review of the previous work done by many researchers has been listed. The objective of the present work and the outline of present study has been detailed in this chapter.

In chapter 3, the detailed explanation of the theory and the detailed formulation related to the present study and the procedure to analyse the buckling behaviour of cross ply laminated composite twisted plate with holes subjected to loading is given in detail. ANSYS software is used to do the analysis. A brief description of the analysis is shown in this chapter.

In chapter 4, the results obtained in this study are discussed in detail and different tables and graphs are presented for better understanding of the behaviour. The effect of aspect ratio, angle of twist, number of layers, cutout size, cutout location, width to thickness ratio on buckling behaviour has been studied and the results thus obtained are present in this chapter.

Finally in chapter 5, by considering the results obtained in this study, the conclusions have been drawn.

CHAPTER 3

3.1 Theory and formulation

A small element of the twisted panel is shown in figure 3.1. In the figure, Φ = angle of twist, a and b are length and width of the plate respectively and h is the thickness of the plate.

The internal forces and moments acting on the element are shown in figure 3.2. The internal forces are N_x, N_y, and N_{xy}, shearing forces are Q_x and Q_y and the moment resultants are M_x, M_y and M_{xy}.

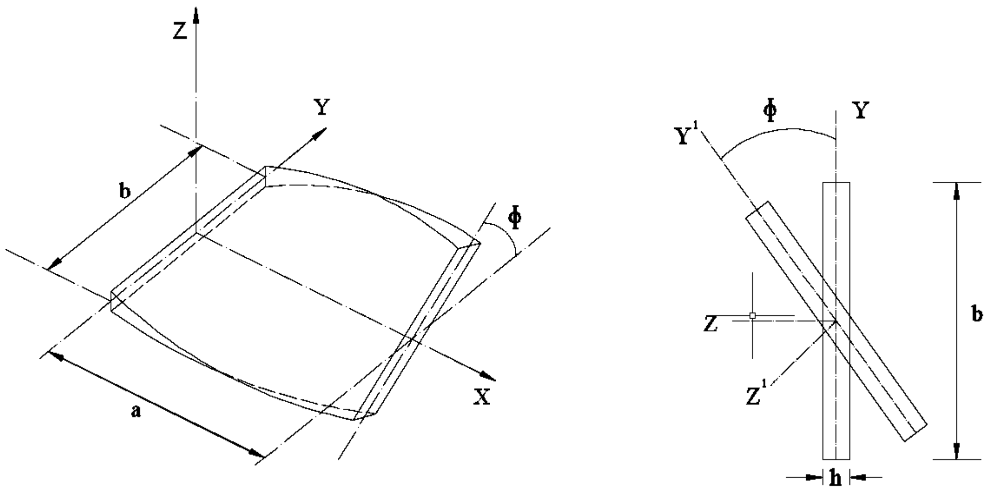

Figure 3.1 Laminated composite twisted plate

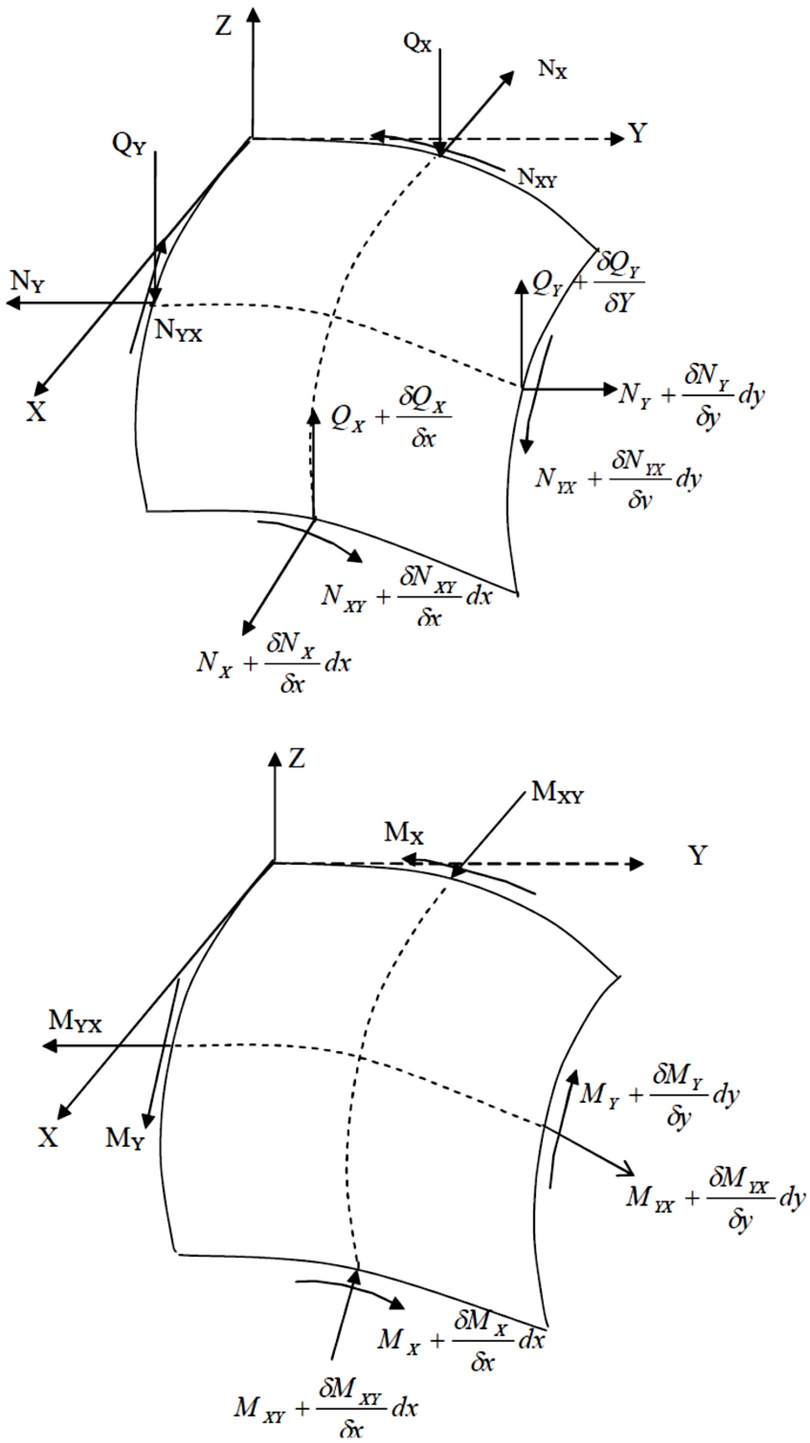

Figure 3.2 Stress resultants in a twisted shell panel

The equations of motion for a doubly curved pretwisted panel subjected to external in-plane loading can be expressed as [6], [17]:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial y} + \frac{Q_x}{R_x} + \frac{Q_y}{R_{xy}} = P_1 \frac{\partial^2 u}{\partial t^2} + P_2 \frac{\partial^2 \theta_x}{\partial t^2}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial x} + \frac{Q_y}{R_y} + \frac{Q_x}{R_{xy}} = P_1 \frac{\partial^2 v}{\partial t^2} + P_2 \frac{\partial^2 \theta_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_x}{R_y} - 2\frac{N_{xy}}{R_{xy}} + N_x^0 \frac{\partial^2 w}{\partial x^2} + N_y^0 \frac{\partial^2 w}{\partial y^2} = P_1 \frac{\partial^2 w}{\partial t^2} \qquad (3.1)$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = P_1 \frac{\partial^2 \theta_x}{\partial t^2} + P_2 \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = P_1 \frac{\partial^2 \theta_y}{\partial t^2} + P_2 \frac{\partial^2 v}{\partial t^2}$$

N_x^0 and N_y^0 are the external loading in the X and Y directions respectively. The constants R_x, R_y and R_{xy} are the radii of curvature in the x and y directions and the radius of twist.

$$(P_1, P_2, P_3) = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} (\rho)_k (1, z, z^2) dz \qquad (3.2)$$

where n = number of layers of the laminated composite twisted curved panel and $(\rho)_k$ = mass density of k_{th} layer from the mid-plane.

3.2 Strain Displacement Relations

The strain-displacement relations are those of Green-Lagrange. The elastic stiffness matrix is derived from the linear part of the strain. The linear strain displacement relations taking into account angle of twist are:

$$\xi_{xl} = \frac{\partial u}{\partial x} + \frac{w}{R_x} + zk_x$$

$$\xi_{yl} = \frac{\partial v}{\partial y} + \frac{w}{R_y} + zk_y$$

$$\gamma_{xyl} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} + zk_{xy} \tag{3.3}$$

$$\gamma_{xzl} = \frac{\partial w}{\partial x} + \theta_x - \frac{u}{R_x} - \frac{v}{R_{xy}}$$

$$\gamma_{yzl} = \frac{\partial w}{\partial y} + \theta_y - \frac{v}{R_y} - \frac{u}{R_{xy}}$$

Where the bending strains are expressed as

$$k_x = \frac{\partial \theta_x}{\partial x},\ k_y = \frac{\partial \theta_y}{\partial y}$$

$$k_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{3.4}$$

The linear strains can be expresses in term of displacements as:

$$\{\varepsilon\} = [\mathrm{B}]\{d_e\} \tag{3.5}$$

Where

$$\{d_e\} = \{u_1 v_1 w_1 \theta_{x1} \theta_{y1} \ldots\ldots\ldots\ldots\ldots\ldots . u_8 v_8 w_8 \theta_{x8} \theta_{y8}\} \tag{3.6}$$

$$[\mathrm{B}] = [[B_1], [B_2] .. \ \ldots\ldots\ldots\ldots\ldots\ldots . [B_8]] \tag{3.7}$$

$$[B_i]=\begin{bmatrix} N_{i,}x & 0 & \frac{N_i}{R_x} & 0 & 0 \\ 0 & N_{i,}y & \frac{N_i}{R_y} & 0 & 0 \\ N_{i,}y & N_{i,}x & 2\frac{N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,}x & 0 \\ 0 & 0 & 0 & 0 & N_{i,}y \\ 0 & 0 & 0 & N_{i,}y & N_{i,}x \\ 0 & 0 & N_{i,}x & N_i & 0 \\ 0 & 0 & N_{i,}y & 0 & N_i \end{bmatrix} \qquad (3.8)$$

The geometric stiffness matrix is derived from the nonlinear part of the strain.

3.3 Stress-strain relations

The composite twisted curved panel is made of thin composite material laminates. The material of every lamina is composed of parallel, continuous fibers of one material installed in a matrix material. Every layer may be considered as homogeneous and orthotropic. A laminated shell element is shown in figure 3.3 where the principle material axes are 1 and 2 and moduli of elasticity of lamina along these directions are E_{11} and E_{22} respectively.

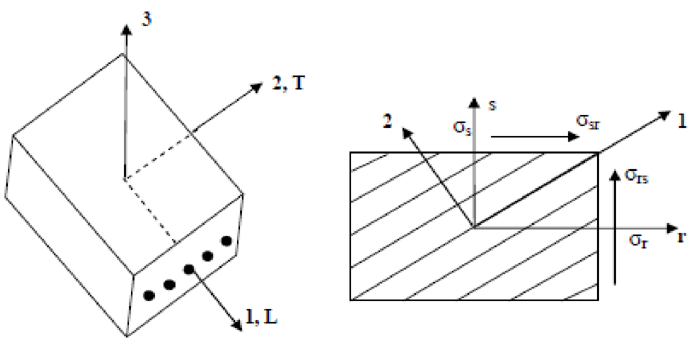

Figure 3.3 Laminated shell element showing principal axes and laminate directions

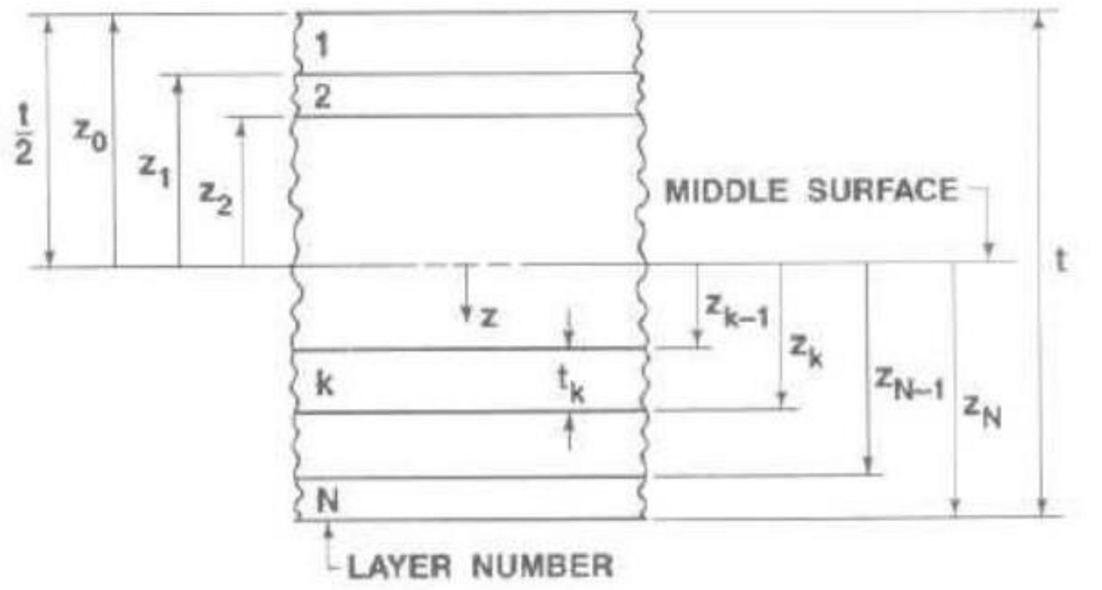

Fig. 3.4 Geometry of a N-layered laminate

The stress-strain relations are:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0\\ 0 & 0 & 0 & 0 & Q_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix} \tag{3.9}$$

Where

$$Q_{11}= \frac{E_{11}}{(1-v_{12}v_{21})} \qquad Q_{12}= \frac{E_{11}v_{21}}{(1-v_{12}v_{21})}$$

$$Q_{21}= \frac{E_{22}}{(1-v_{12}v_{21})} \qquad Q_{22}= \frac{E_{22}}{(1-v_{12}v_{21})} \tag{3.10}$$

$$Q_{66}= G_{12} \qquad Q_{44}= \mathrm{k}G_{13}$$

$$Q_{55}= kG_{23}$$

The on –axis elastic constant matrix corresponding to the fiber direction is given by

$$[Q_{ij}] = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & & Q_{55} \end{bmatrix} \tag{3.11}$$

If the arbitrary principal axes makes an angle Θ with the material principal axes, then the elastic constant matrix for this case is obtained by coordinate transformation. The off-axis elastic constant matrix is given as

$$[\bar{Q}_{ij}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \tag{3.12}$$

$$[\bar{Q}_{ij}] = [T]^T][Q_{ij}][T] \tag{3.13}$$

[T] is the transformation matrix. After transformation the elastic stiffness coefficients are:

$$\bar{Q}_{11} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4)$$

$$\bar{Q}_{22} = Q_{11}n^4 + 2(Q_{12} + Q_{66})m^2n^2 + Q_{22}m^4 \tag{3.14}$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3m$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 + Q_{66}(n^4 + m^4)$$

For transverse shear deformation, the elastic constant matrix is

$$\bar{Q}_{44} = G_{13}m^2 + G_{23}n^2$$

$$\bar{Q}_{45} = (G_{13} - G_{23})mn \tag{3.15}$$

$$\bar{Q}_{55} = G_{13}n^2 + G_{23}m^2$$

Where m = cosθ and n = sinθ

Therefore the constitutive relations are $\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$ (3.16)

By integrating through the thickness, the force and moment resultants are obtained as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_Y \\ M_{xY} \\ Q_x \\ Q_Y \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_x \\ \tau_{xy} \\ \sigma_x z \\ \sigma_{yz} \\ \tau_{xy} z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{3.17}$$

Where σ_x, σ_y are the normal stresses along X and Y directions, τ_{xy},τ_{xz} and τ_{yz} are shear stresses in xy, xz and yz planes respectively.

The constitutive relationships for bending transverse shear of a doubly curved shell becomes

$$\begin{Bmatrix} N_x \\ N_P \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{112} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{3.18}$$

This can also be stated as $\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix}$ (3.19)

Or $\{F\} = [D]\{\varepsilon\}$ (3.20)

Where A_{ij}, B_{ij}, D_{ij} and S_{ij} are the extensional , bending-stretching coupling, bending and transverse shear stiffness. They may be defined as:

$$A_{ij} = \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^3 - z_{k-1}^3) \; ; i, j = 1,2,6 \qquad (3.21)$$

$$S_{ij} = k \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1}) \; ; i, j = 4,5$$

And k is the transverse shear correction factor.

3.4 Methodology

In this project, the models of a laminated composite twisted plate and flat plate with holes subjected to axial loading are developed. The first step is to develop an ANSYS model of a laminated composite plate. The laminated composite flat plate with holes will be subjected to axial compression and buckling load obtained compared with the previous results. Then a laminated composite twisted plate with holes is studied for its stability characteristics with in-plane loads.

ANSYS methodology:

ANSYS 15.0 version software is used. There are three stages for solving a problem in ANSYS:

Preprocessing:

This is where the problem is defined.

1. Define lines/areas/volumes

2. Characterize component type and material/geometric properties

3. Mesh lines/area/volumes as required

Solution

Here we specify the loads (point or pressure), constraints (translational and rotational) and finally solve the resulting set of equations.

Post-processing

In this stage, further processing and viewing the results is done like reading values of nodal displacements, values of element forces and moments, Deflection plots, Stress contour diagrams, etc

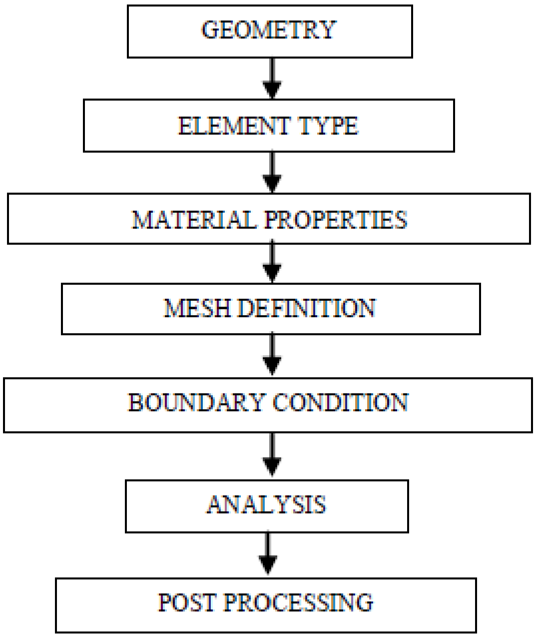

Fig. 3.5 Solution steps in ANSYS

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Introduction

The composite plates are widely used as structural elements in aerospace, civil, mechanical and other structures. These structures are generally subjected under severe dynamic loading and different conditions during their service life. Hence, it is the designer's quest to model these complex structural problems precisely and predict the deflections and other responses with less computational effort.

The proposed model has been produced taking into account the limited component steps as in ANSYS and solved using the APDL coding. Shell 281 element has been utilized in ANSYS. To demonstrate the adequacy of the present model, convergence test has been done. Based on the convergence, buckling values which are obtained are compared with the published results. The influences of various parameters on the buckling analysis of laminated composite plates with holes are generated by solving some new examples.

This chapter presents first a convergence study to determine the mesh size required for further analysis. Then a comparative study is done to validate the ANSYS modelling. After these studies, the present problem is studied in detail.

4.2 Convergence Study

The convergence study is first done for the mesh size necessary for the buckling analysis of the square laminated composite twisted cantilever plates with an angle of twist of 10° for different mesh sizes and is shown in Table 4.1. Based on this study, an 8 x 8 mesh was chosen for solving the problem. In all cases

a = length of plate

b= width of plate

h= thickness of plate

Table 4.1 Convergence of Non- dimensional Buckling Load (λ) of laminated composite twisted plate with varying mesh size

E_{11} = 141 GPa. , E_{22} = 9.23 GPa, υ_{12} =0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa

a/b =1, b/h=250, Ø=10^0 , [0/ 90^0] plate

Non-dimensional buckling load is given by $\lambda = \frac{N_x b^2}{E_{22} h^3}$

Mesh size	Angle of twist Ø	Buckling load (N/m)	Non-dimensional buckling load(λ)
4×4	10°	206.93	0.70060
6×6	10°	206.89	0.70046
8×8	10°	206.89	0.70046
Sahu *et al.*[16]	10°	205.24	0.69490

Table 4.2 Convergence of Non- dimensional Buckling Load (λ) of laminated composite twisted plate with varying mesh size

E_{11} = 141 GPa. , E_{22} = 9.23 GPa, υ_{12} =0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa

a/b =1, b/h=250, Ø=10^0 , [0/ 90^0/0/90^0] plate

Non-dimensional buckling load is given by $\lambda = \frac{N_x b^2}{E_{22} h^3}$

Mesh size	Angle of twist	Buckling load (N/m)	Non-dimensional buckling load(λ)
4×4	10°	419.19	1.41920
6×6	10°	419.14	1.41906
8×8	10°	419.13	1.41903
Sahu *et al.*[16]	10°	415.81	1.40780

4.3 Comparison with previous studies

In this study, the accuracy and efficiency of the solution are established through comparison with previous studies. The lowest non-dimensional buckling load of a square laminated composite twisted cantilever plate obtained by ANSYS is compared with those obtained by Sahu and Asha [2008] in Table 4.3.

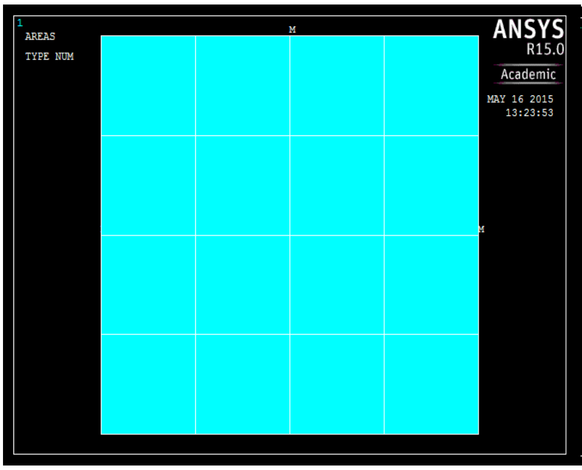

Figure 4.1 (a) 4×4 Meshing

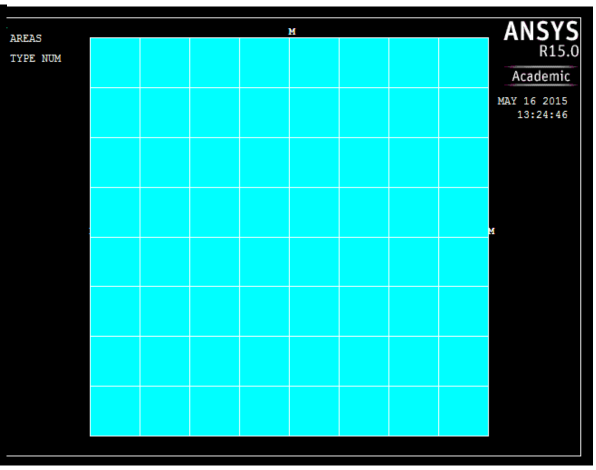

Figure 4.1 (b) 8×8 Meshing

Figure 4.1 Meshing of the plate

Table 4.3 Comparison of non-dimensional buckling load for cross-ply plates with different ply lay-ups with previous study

a/b = 1, b/h= 250

Angle of twist Ø	**Cross ply laminates**	**Non dimensional buckling load (λ)**	
		Sahu *et al.*[16]	**Present study (ANSYS)**
0°	$[0/90^0]$	0.7106	0.7106
	$[0/90^0/0/90^0]$	1.4432	1.4327
	$[0/90^0/90^0/0]$	2.7891	2.7890
	$[0/90^0/0/90^{0/}0/90^0/0/90^0]$	1.6254	1.6220
10°	$[0/90^0]$	0.6949	0.7004
	$[0/90^0/0/90^0]$	1.4078	1.4190
	$[0/90^0/90^0/0]$	2.7273	2.7491
	$[0/90^0/0/90^{0/}0/90^0/0/90^0]$	1.5860	1.5987
20°	$[0/90^0]$	0.6473	0.6695
	$[0/90^0/0/90^0]$	1.3114	1.3368
	$[0/90^0/90^0/0]$	2.5405	2.5784
	$[0/90^0/0/90^{0/}0/90^0/0/90^0]$	1.4774	1.4985
30°	$[0/90^0]$	0.5689	0.5896
	$[0/90^0/0/90^0]$	1.1526	1.1648
	$[0/90^0/90^0/0]$	2.2329	2.2400
	$[0/90^0/0/90^{0/}0/90^0/0/90^0]$	1.2985	1.3136

From above table, the results are quite comparable.

Table 4.4 Comparison of buckling load for cross-ply plates with different ply lay-ups with previous study

E_{11} = 141 GPa. , E_{22} = 9.23 GPa, υ_{12} =0.313, G_{12} = 4.95GPa, G_{23} = 2.96GPa

Cross ply layup	**Buckling load (N/m)**							
	b/h=25		b/h=50		b/h=100		b/h=250	
	Present study ANSYS	Sahu *et al.* (2008)	Present study ANSYS	Sahu *et al.* (2008)	Present study ANSYS	Sahu *et al.* (2008)	Present study ANSYS	Sahu *et al.* (2008)
[0/90^0]	202490	198626.08	25365	24895.47	3172.6	3114.36	203.09	199.38
[0/ 90^0/0/90^0]	408100	401251.03	51321	50400	6424.3	6308.38	411.45	403.94
[0/ 90^0/90^0/0]	786120	772946.51	99284	97499.37	12444	12217.06	797.09	782.54
[0/90^0/0/90^0/0/ 90^0/0/90^0]	459910	451723.41	57822	56770.53	7238.9	7106.66	463.54	455.08

From the above table 4.4 we can observe that the results are quite comparable.

4.4 Numerical results:

4.4.1 Flat laminated composite plates:

4.4.1.1 Variation of buckling load with different hole diameter:

Mechanical properties of the plate taken:

E_{11} = 141 GPa, E_{22} = 9.23 GPa, $\upsilon_{12} = \upsilon_{13}$ = 0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa

A square plate with length and breadth equaling 0.5m is considered. The diameter of the hole is found from the different d/b ratio. The total thickness is taken to be 2mm. A central circular hole is taken for the analysis.

Non-dimensional buckling load is given by $\lambda = \frac{N_x b^2}{E_{22} h^3}$

Table 4.5 Variation of non-dimensional buckling load for laminated composite cantilever plate with a hole with different d/b ratios.

d/b ratio	**Non dimensional buckling load (λ)**		
	[0/90^0]	[0/ 90^0/0/90^0]	[0/90^0/0/90$^{0/}$0/90^0/0/90^0]
0.1	0.7597	1.2406	1.3413
0.2	0.7397	1.2275	1.3393
0.3	0.7317	1.1890	1.2963
0.4	0.7038	1.0933	1.1664

From the above table 4.5 we can observe that as the hole diameter increases the non-dimensional buckling load decreases for all ply lay ups. Figure 4.1 shows a clear picture of variation.

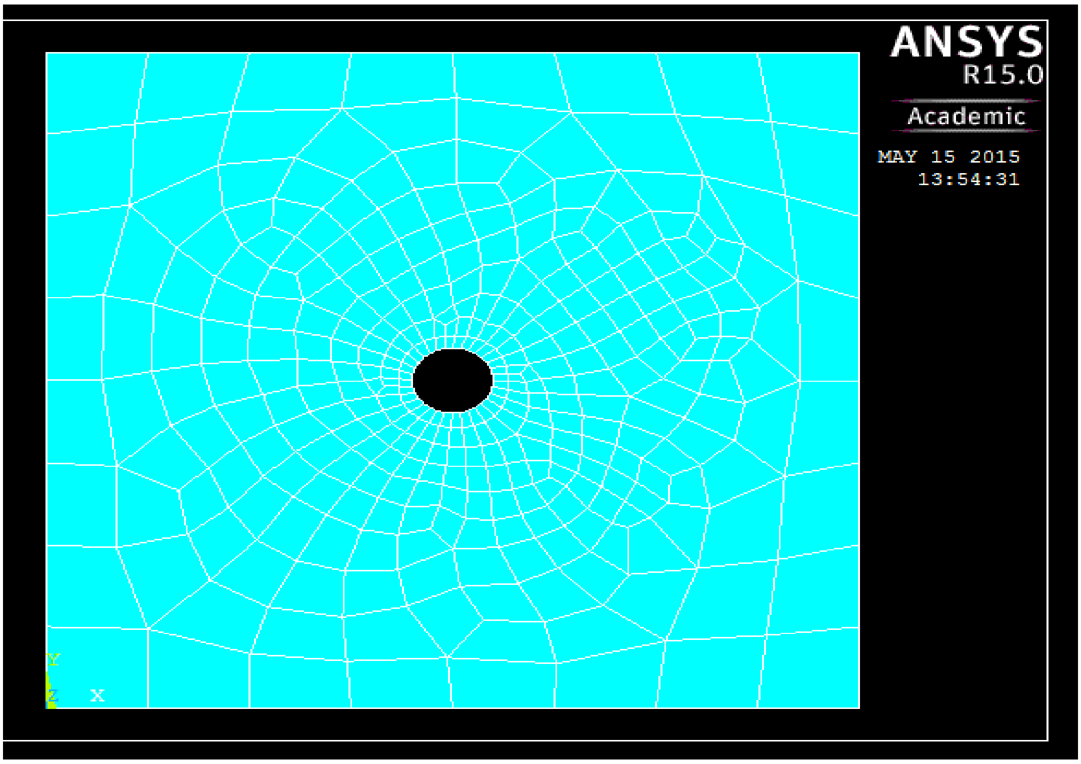

Figure 4.2 Meshing of flat plate with a hole

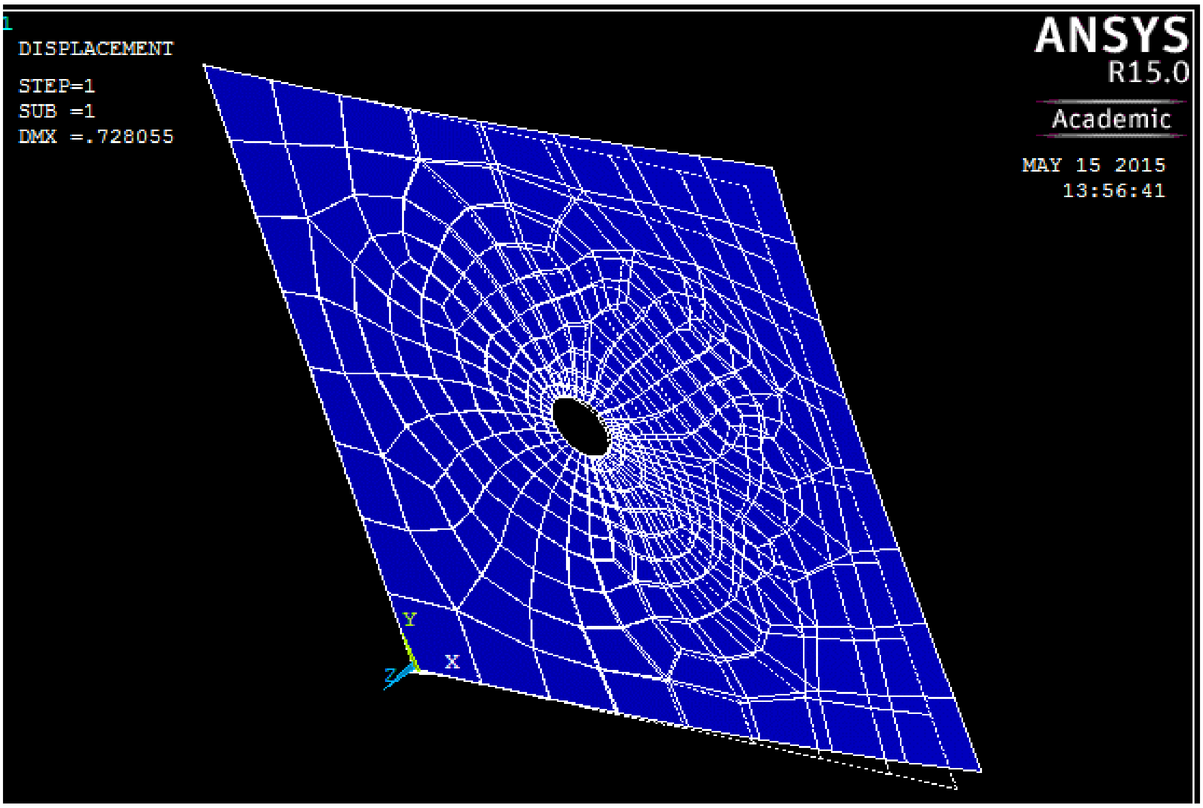

Figure 4.3 Deformed shape of a flat cantilever plate with a hole

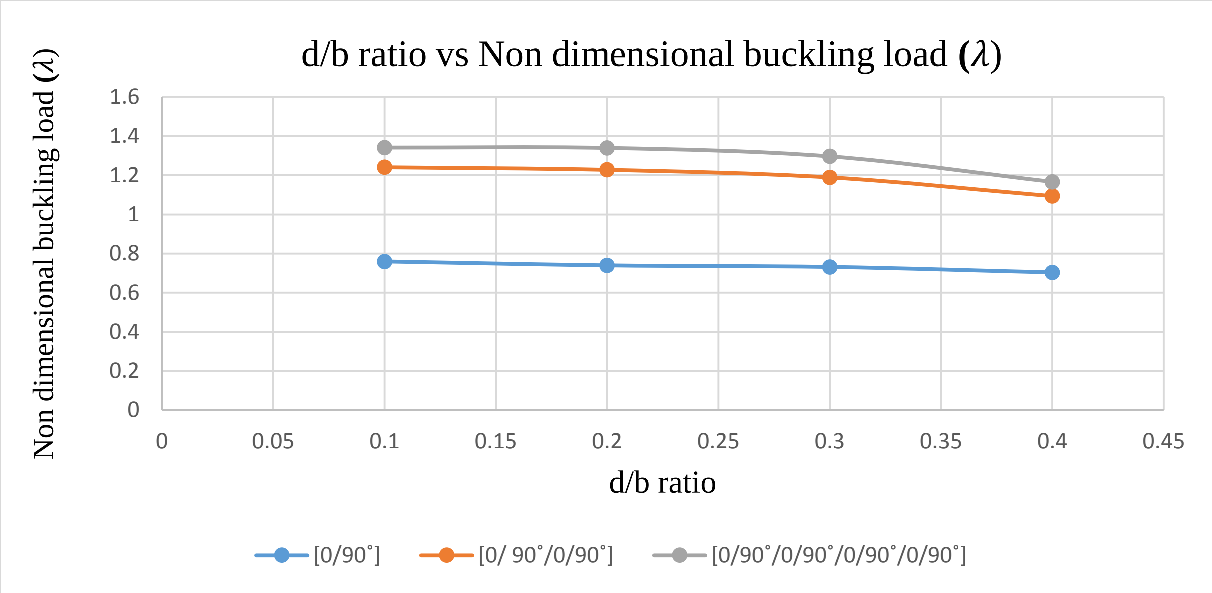

Figure 4.4: Variation of buckling load with d/b ratio of hole for flat cross-ply composite plates

4.4.1.2 Variation of buckling load with different d/b ratios for different ply lay ups:

Mechanical properties of the plate taken:

E_{11} = 130 GPa , E_{22} = 10 GPa, E_{33} = 10 GPa

G_{12} = G_{12} = 5 GPa,

υ_{12} = υ_{13} = υ_{23} = 0.35

Dimension of the plate taken:

A square plate with length and breadth equaling 2 m is considered. The diameter of the hole is found from the different d/b ratio. The thickness of each layer is 1.5 mm.

Non-dimensional buckling load is given by $\lambda = \frac{N_x b^2}{E_{22} h^3}$

Table 4.6 Variation of non-dimensional buckling load for simply supported laminated composite plate with different d/b ratios.

d/b ratio	Non dimensional buckling load (λ)		
	$[0/90^0]_4$	$[45^0/-45^0]_4$	$[30^0/-30^0]_4$
0	14.8	35	32.80
0.1	14.53	29.58	24.46
0.2	12.41	29.07	23.95
0.3	10.69	28.47	22.51
0.4	9.94	24.99	21.53

From the above table we can observe that as the d/b ratio increases, the non-dimensional buckling load decreases for different ply lay ups. Figure 4.2 shows a clear picture of variation.

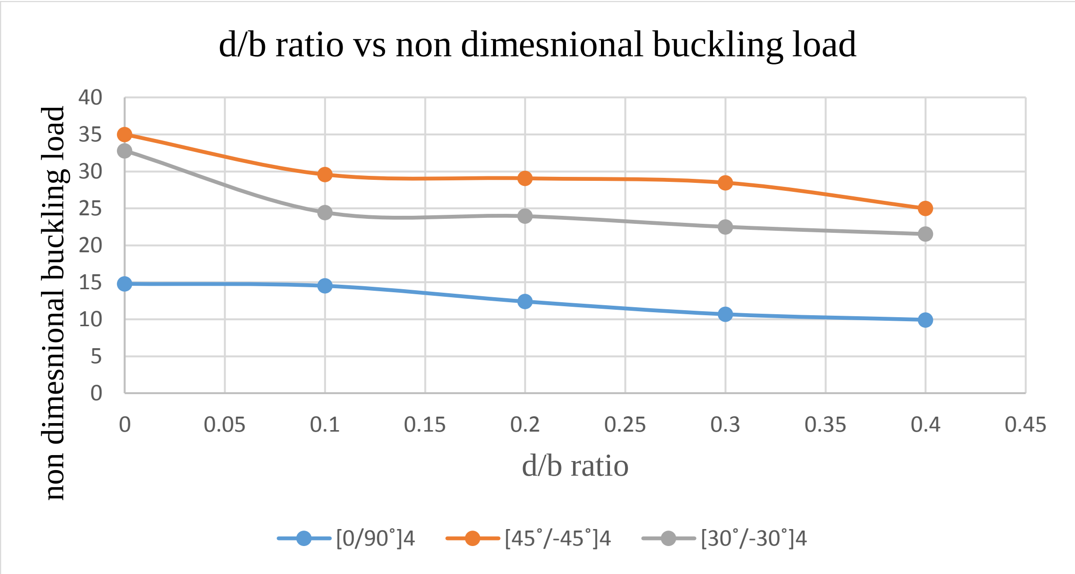

Figure 4.5 : Variation of buckling load with d/b ratio of hole for flat composite plates for different ply lay-ups

The location of the hole is also varied and the study is shown in Table 4.7. The diameter of the hole is varied too.

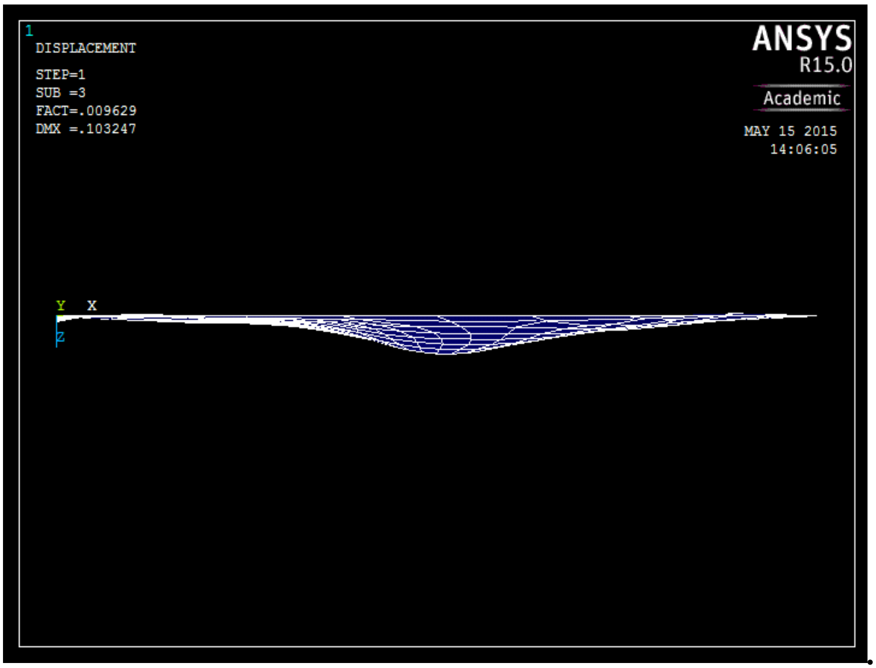

Figure 4.6 Deformed shape a simply supported laminated composite plate with a hole when uniaxial compressive loading is applied.

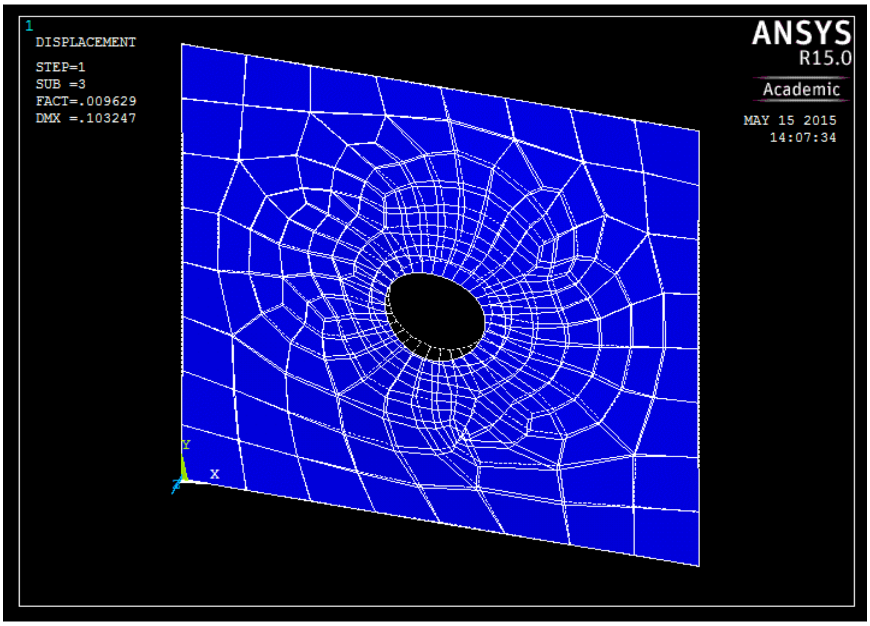

Figure 4.7 Deformed shape a simply supported laminated composite plate with a hole when uniaxial compressive loading is applied.

Table 4.7 Variation of non-dimensional buckling load for simply supported laminated composite plate with different d/b ratios with different ex/b ratio..

$[45/45/90/0]_s$ plate was taken

e_x / b	Non dimensional buckling load (λ)				
	d/b = 0.1	d/b = 0.2	d/b = 0.3	d/b = 0.4	d/b = 0.5
0.5	22.46	19.84	16.14	15.78	14.94
0.4	22.55	19.86	15.6	13.44	12.87
0.3	22.90	19.97	14.87	11.4	10.54
0.2	23.25	20.82	16.78		
0.1	23.91				

From the table 4.7, we can observe that the non-dimensional buckling load decreases for d/b=0.4 and 0.5 as the hole moves from center towards the edges of the plate and the non dimensional buckling load increases for d/b = 0.1 and 0.2 as the hole moves towards the edge of the plate as shown in the figure.

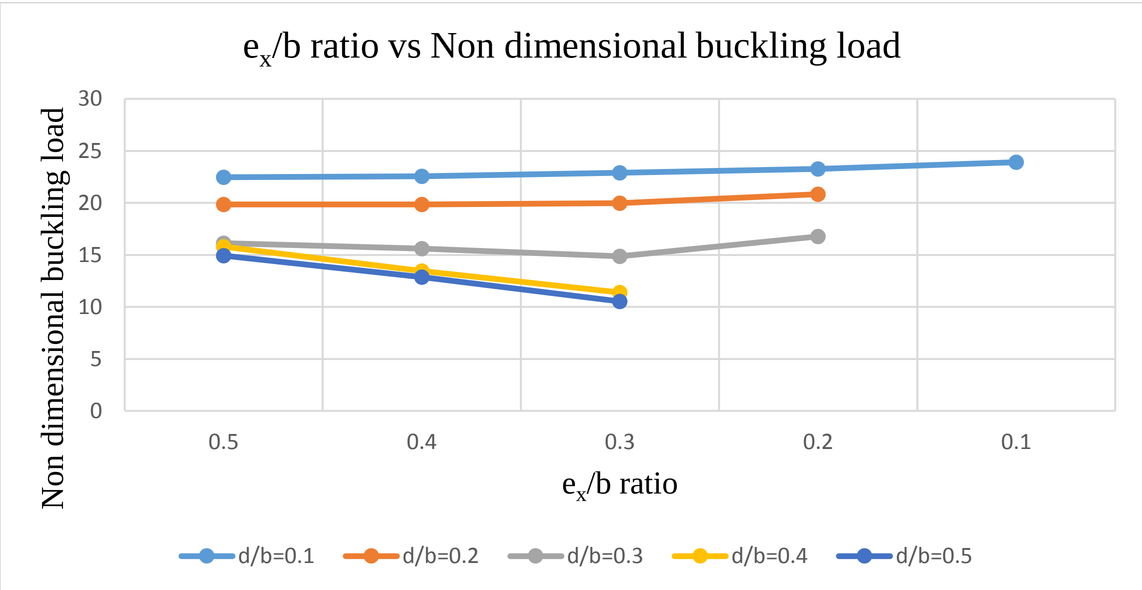

Figure 4.8 : Variation of buckling load with d/b ratio of hole and location of hole for flat composite plates

4.4.2 ANALYSIS OF TWISTED PLATED WITH A HOLE:

4.4.2.1 Variation of non-dimensional buckling load with different d/b ratios:

Mechanical properties of the plate taken:

E_{11} = 141 GPa, E_{22} = 9.23 GPa,

$\upsilon_{12} = \upsilon_{13}$ = 0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa

Dimension of the plate taken:

It is a square plate with length and breadth equaling 0.5m. The diameter of the hole is found from the different d/b ratio. The total thickness is taken to be 2mm. A central circular hole is taken for the analysis.

Table 4.8 Variation of non-dimensional buckling load for laminated composite twisted cantilever plate with a hole with different d/b ratios.(Ø=10^0)

d/b ratio	Non dimensional buckling load (λ)		
	[0/90^0]	[0/ 90^0/0/90^0]	[0/90^0/0/$90^{0/}$0/90^0/0/90^0]
0.1	0.7293	1.2331	1.3760
0.2	0.7132	1.1950	1.3056
0.3	0.7079	1.1793	1.2648
0.4	0.6943	1.0433	1.1411

From the above table 4.5 we can observe that as the hole diameter increases the non dimensional buckling load decreases for all ply lay ups. Figure 4.1 shows a clear picture of variation.

.

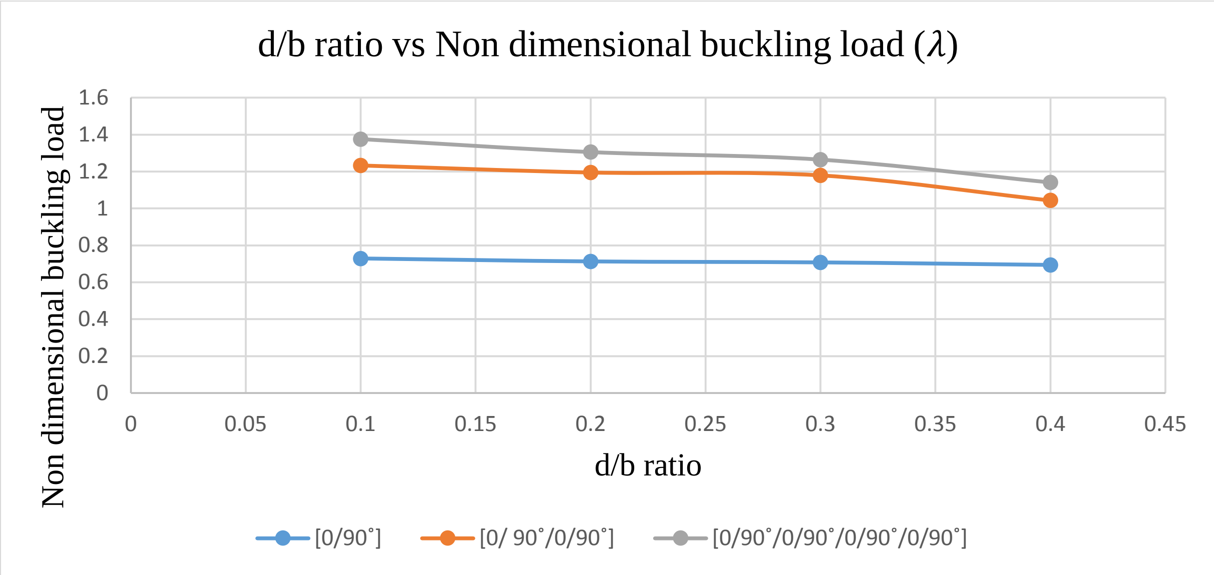

Figure 4.9 : Variation of buckling load with d/b ratio of hole for twisted composite plates

Table 4.9 Variation of non-dimensional buckling load for laminated composite twisted cantilever plate with a hole with different twist angle(Ø)

[0/ 90^0/0/90^0] plate was taken

Angle of twist (Ø)	**Non dimensional buckling load (λ)**			
	d/b = 0.1	**d/b = 0.2**	**d/b = 0.3**	**d/b = 0.4**
0˚	1.2406	1.2275	1.1890	1.0933
10˚	1.2331	1.1950	1.1793	1.0433
20˚	1.1861	1.1761	1.1652	1.0208
30˚	1.1725	1.1645	1.1380	0.9867

From the above table 4.9 we can observe that as the angle of twist increases, the non-dimensional buckling load decreases for a particular hole diameter. Figure 4.5 shows a clear picture of variation.

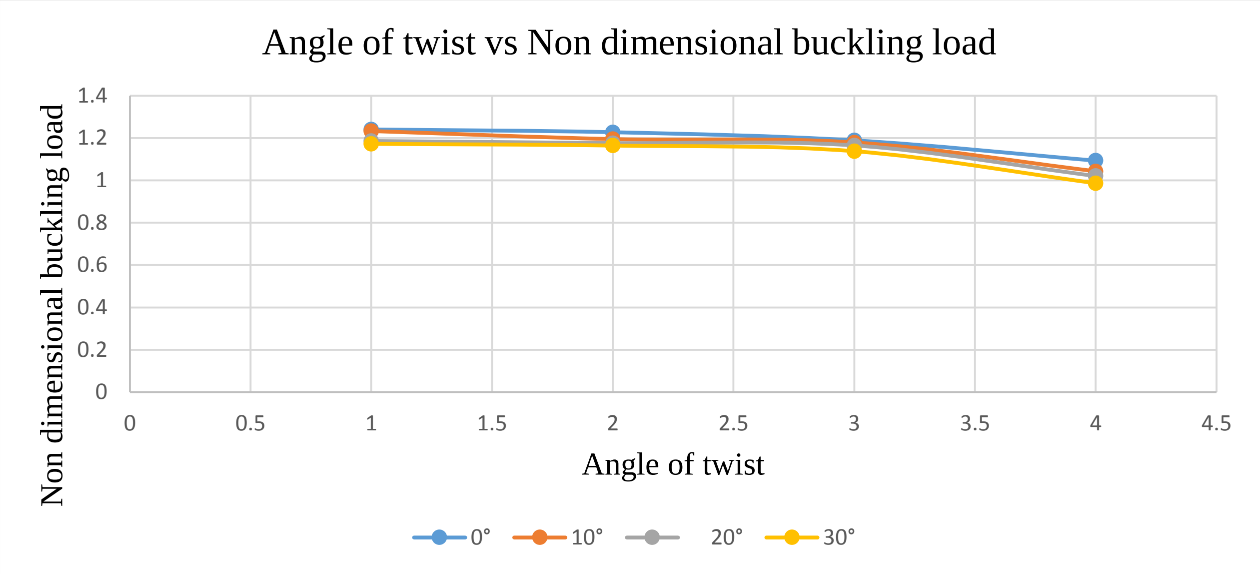

Figure 4.10 : Variation of buckling load with d/b ratio of hole and angle of twist for twisted composite plates

4.4.2.2 Variation of non-dimensional buckling load with different b/h ratios

Mechanical properties of the plate taken:

$E_{11} = 141$ GPa, $E_{22} = 9.23$ GPa,

$\upsilon_{12} = \upsilon_{13} =$ 0.313, $G_{12} = 5.95$GPa, $G_{23} = 2.96$GPa

Dimensions of the plate taken:

A square plate with length and breadth equaling 0.5m is considered. The diameter of the hole is taken as 100mm. The total thickness is taken from different b/h ratios. A central circular hole is taken for the analysis. Angle of twist Ø = 20^0.

Table 4.10 Variation of non-dimensional buckling load for twisted laminated composite cantilever plate with different b/h ratios.

b/h ratio	Non dimensional buckling load (λ)			
	[0/90^0]	[0/ 90^0/90^0/0]	[0/ 90^0/0/90^0]	[0/90^0/0/90$^{0/}$0/90^0/0/90^0]
50	88.0177	147.24	121.81	161.3078
100	11.2490	18.7049	15.9581	20.568
200	1.3653	2.3701	2.1036	2.6122
250	0.6999	1.2189	1.1761	1.3444

From the table 4.10, we can observe that as the b/h ratio increases, that is thickness decreases the non-dimensional buckling load decreases. This is also shown in figure 4.6.

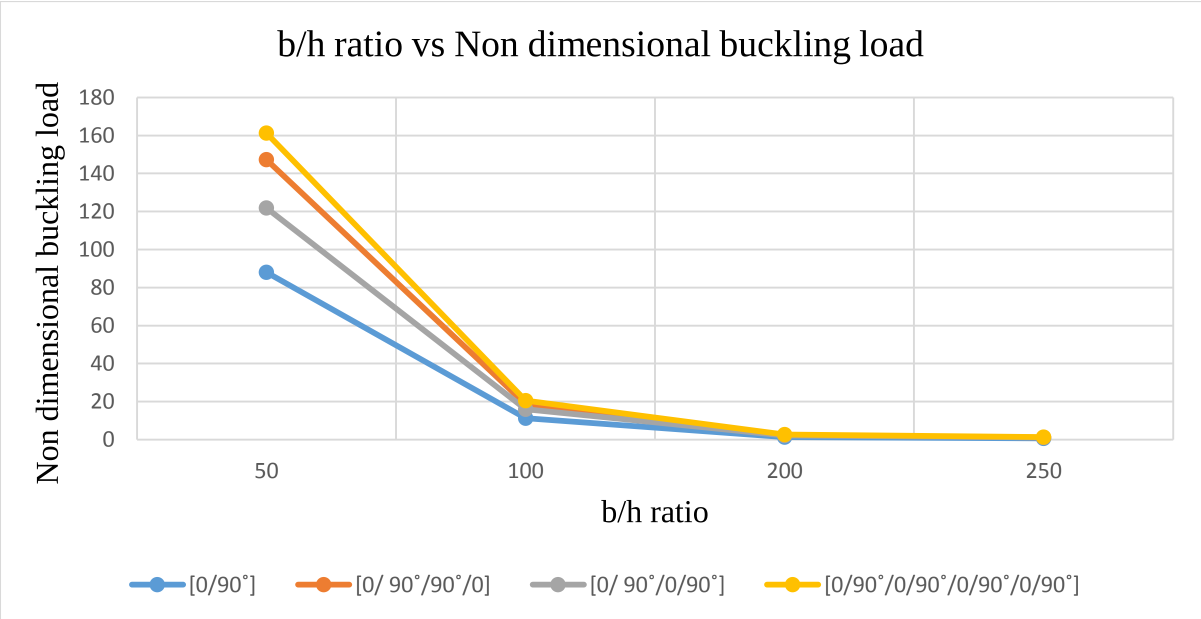

Figure 4.11: Variation of buckling load with b/h ratio and constant hole diameter for twisted composite plates

The analysis is done for a plate with a central hole and results compared for different aspect ratio. The deformed shape is shown in figure.

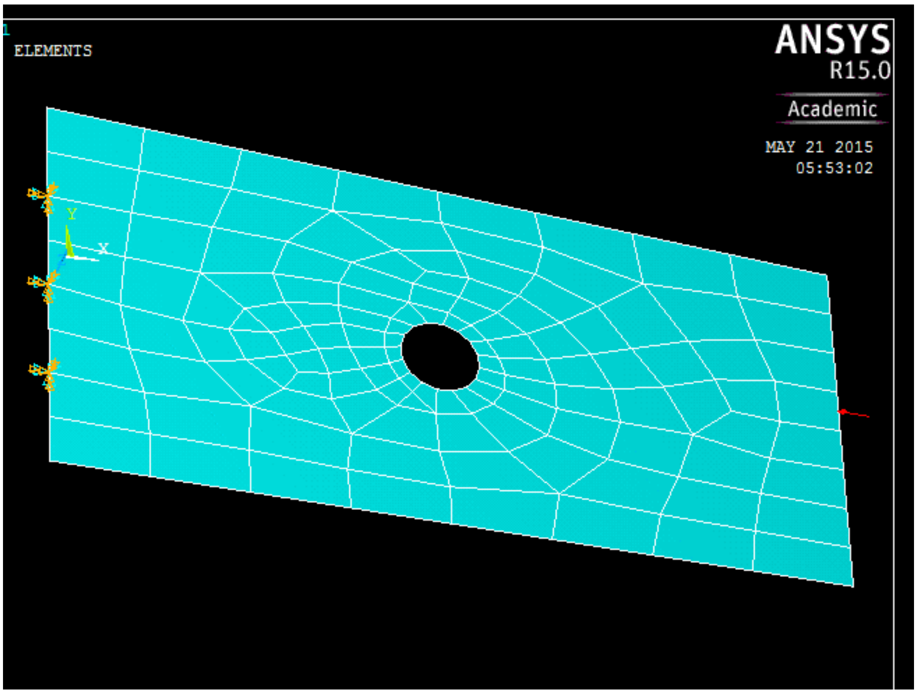

Figure 4.12: Variation of buckling load with b/h ratio and constant hole diameter for twisted composite plates

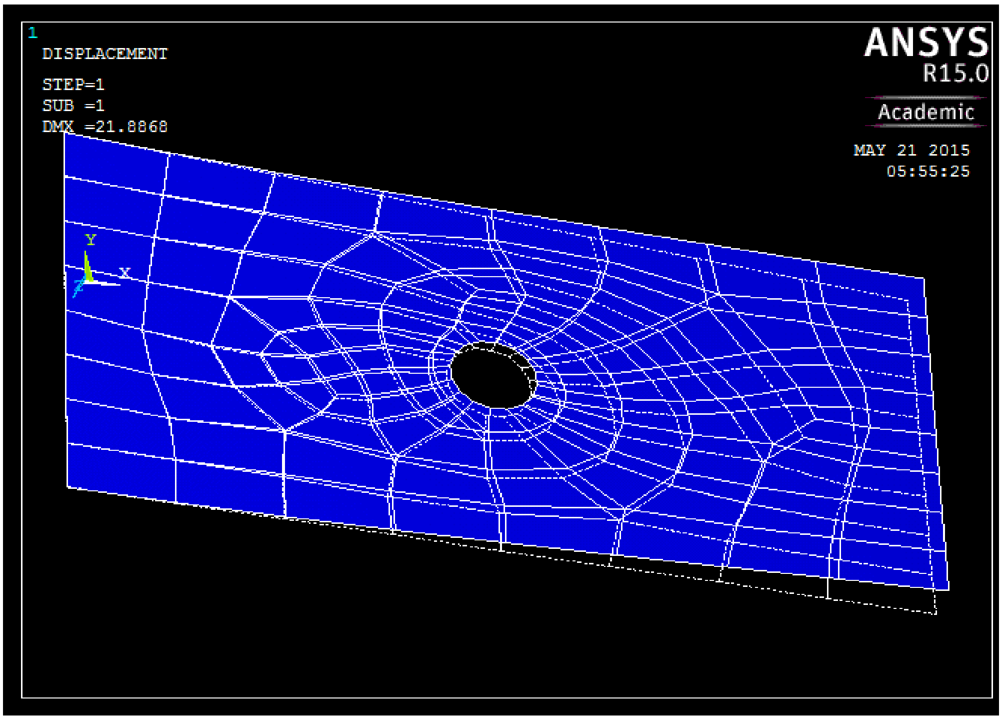

Figure 4.13 Deformed shape of a laminated composite twisted plate with hole

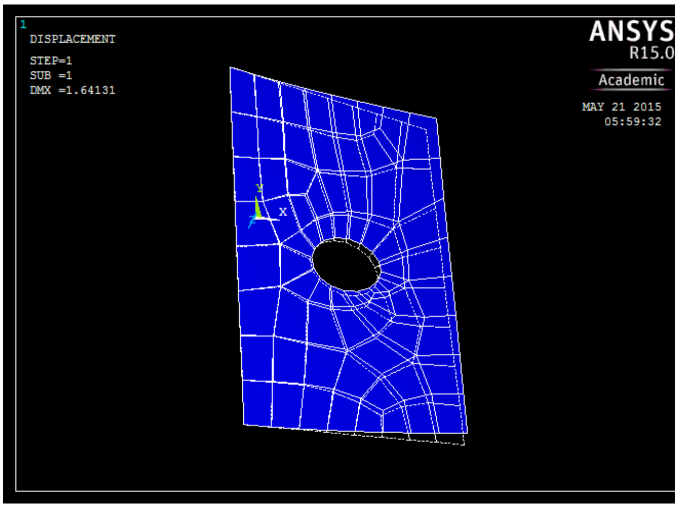

Figure 4.14 Deformed shape of laminated composite twisted plate when subjected to loading

Table 4.11 Variation of non-dimensional buckling load for twisted laminated composite cantilever plate for different a/b ratios and different hole diameters

Angle of twist [Ø] = 10^0 and [$0/90^0$] plate

a/b ratio	Non dimensional buckling load (λ)			
	d/b = 0.1	d/b = 0.2	d/b = 0.3	d/b = 0.4
2	0.1999	0.1990	0.1744	0.1623
1.5	0.3375	0.3286	0.2933	0.2820
1	0.7293	0.7132	0.7079	0.6943
0.5	2.9873	2.7391	2.7279	2.5346

From the table 4.11, we can observe that as the a/b ratio decreases, the non-dimensional buckling load increases for a particular hole diameter.

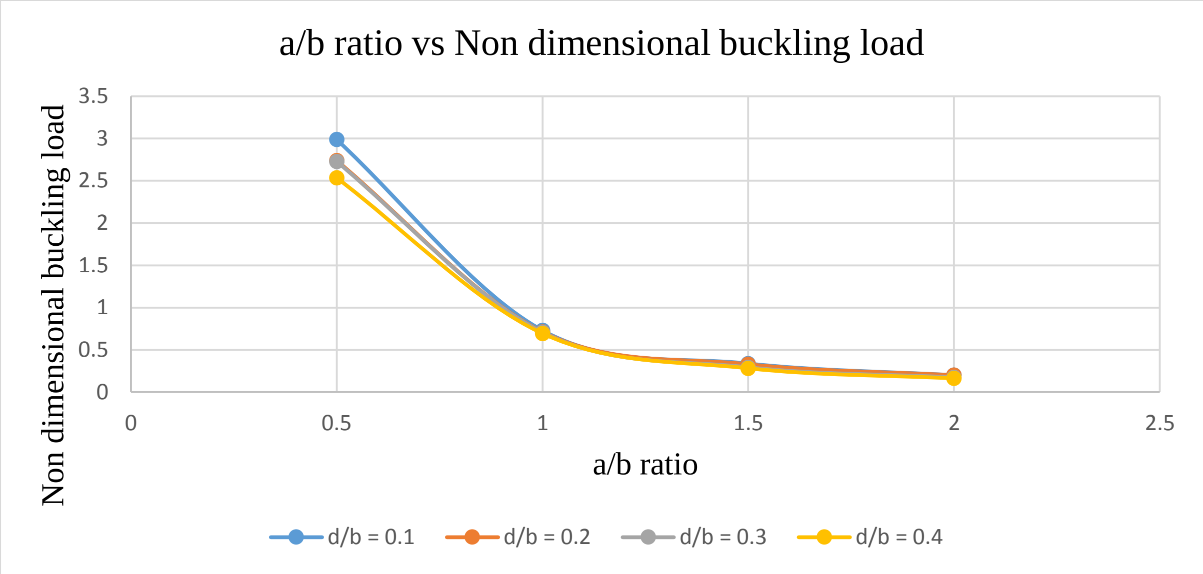

Figure 4.15: Variation of buckling load with a/b ratio and different hole diameters for twisted composite plates

4.4.2.3 Multiple holes in laminated composite twisted plate

The analysis is now carried out for a twisted plate with multiple holes. Two holes are symmetrically placed.

Table 4.12 Variation of non-dimensional buckling load for twisted laminated composite cantilever plate with different a/b ratios with two holes of dia d=100mm

a/b ratio	Non dimensional buckling load(λ)			
	[0/90^0]	[0/ 90^0/0/90^0]	[0/ 90^0/90^0/0]	[0/90^0/0/90$^{0/}$0/90^0/0/90^0]
0.5	2.36433	4.19725	2.86193	4.64271
1	0.67111	1.13102	0.85583	1.24285
1.5	0.31616	0.50711	0.45088	0.55406
2	0.18958	0.29332	0.33968	0.31916

Here again, buckling load decreases as aspect ratio increases as seen in Table 4.12.

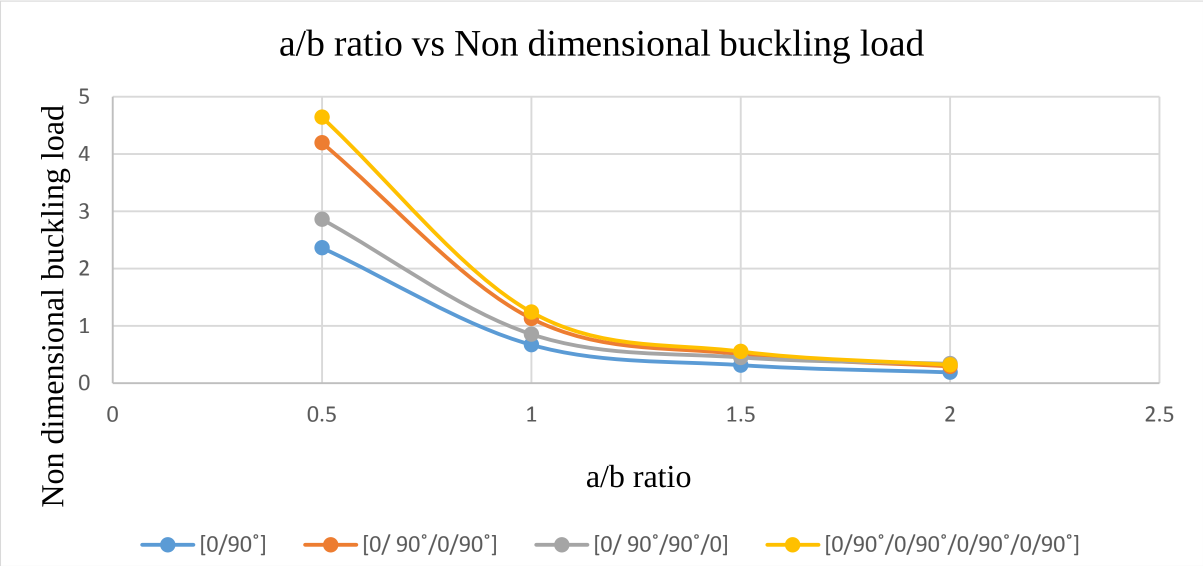

Figure 4.16: Variation of buckling load with a/b ratio for twisted composite plates with two symmetric holes

The location of the holes is changed by varying the e_x/b ratio. The results are shown in Table 4.13.

Table 4.13 Variation of non-dimensional buckling load for twisted laminated composite cantilever plate with different e_x/b ratios for a hole diameter d=100mm

e_x/b ratio	Non dimensional buckling load		
	$[0/90^0]$	$[0/\ 90^0/0/90^0]$	$[0/90^0/0/90^0/0/90^0/0/90^0]$
0.25	0.6890	1.1771	1.2566
0.50	0.7132	1.1950	1.3056
0.75	0.7665	1.3585	1.4529

From table 4.13, as e_x/b ratio increases the buckling load factor increases for different ply layups.

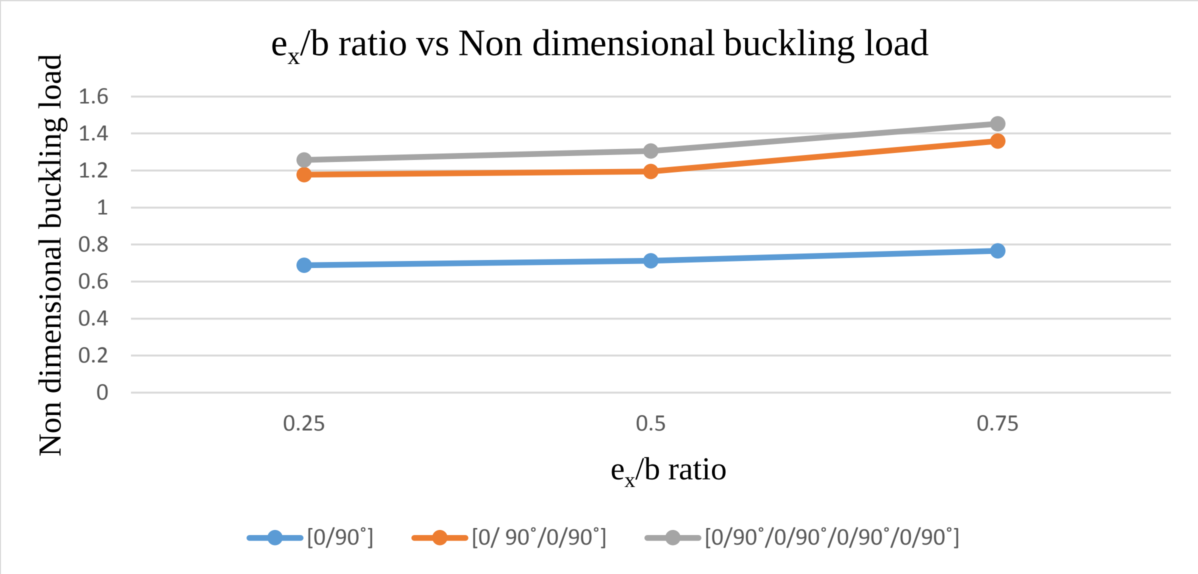

Figure 4.17: Variation of buckling load with e_x/b ratio for twisted composite plates with different ply layups.

A study is also done for both varying e_x/b ratio and d/b ratio. This is shown in Table 4.14.

Table 4.14 Variation of non-dimensional buckling load for twisted laminated composite cantilever plate with different e_x/b ratios for different hole diameters

[0/90^0/0/90$^{0/}$0/90^0/0/90^0] plate was taken

e_x/b ratio	Non dimensional buckling load			
	d/b = 0.1	d/b = 0.2	d/b = 0.3	d/b = 0.4
0.25	1.3241	1.2566	1.1932	0.9704
0.50	1.3760	1.3056	1.2648	1.1411
0.75	1.5049	1.4529	1.4019	1.3543

From table 4.14, as e_x/b ratio increases the buckling load factor increases for different different hole diameter.

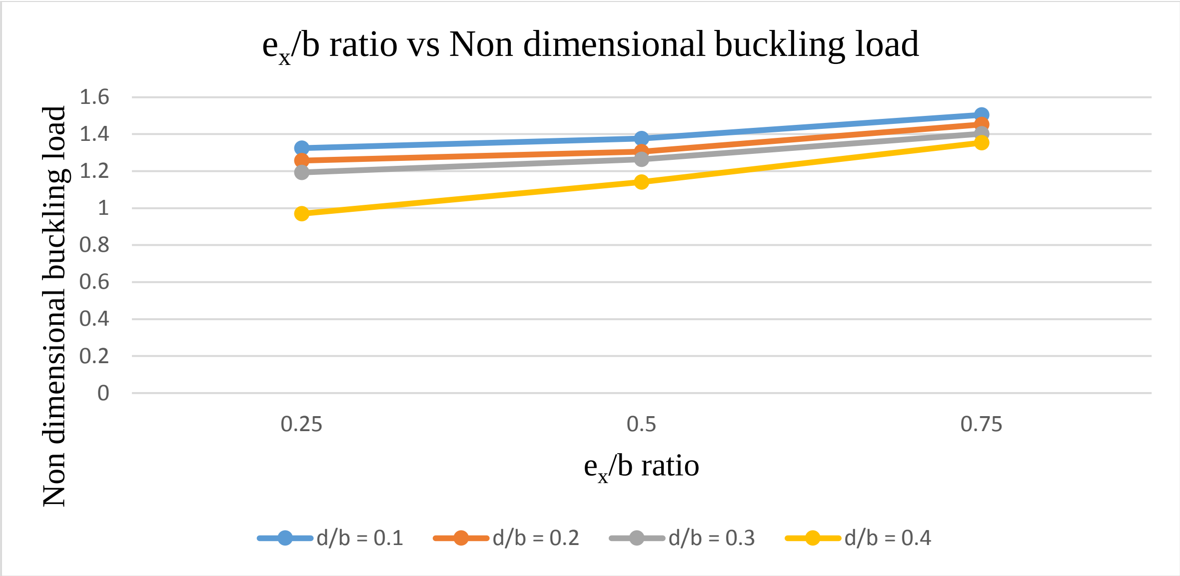

Figure 4.18: Variation of buckling load with e_x/b ratio for twisted composite plates with different hole diameter.

CONCLUSION

In this study, the behavior of square cross ply laminated composite twisted plates with holes subjected to uniaxial compression is studied for its buckling characteristics considering various parameters. The analysis is carried out using ANSYS software. The various parameters considered in this study are aspect ratio, angle of twist, number of layers, cutout size and cutout location. Based on the study it is observed that

1. For a flat laminated composite plate, it was observed that as the hole diameter increases, the non-dimensional buckling load decreases for all ply layups. This was observed for both angle-ply and cross-ply layups.
2. The non-dimensional buckling load decreases as the hole diameter increases for flat laminated composite plate..
3. For a twisted plate with twist angle 10°, it was seen that as the hole diameter increases, the buckling load decreases for all cross-ply layups.
4. For a particular cross-ply lamination scheme, as the angle of twist increases, the buckling load decreases for all hole sizes.
5. As the aspect ratio decreases, the buckling load considered increases for a particular hole diameter. This is also observed for different hole diameters.
6. As the b/h ratio increases, the non-dimensional buckling for different ply layups decreases for a fixed hole diameter.
7. As the diameter of the hole increases, the buckling load of a laminated composite twisted plate decreases for a particular angle of twist.
8. As the hole moves towards the edge from the center of the plate, the buckling load factor a laminated composite twisted plate goes on decreasing.

REFERENCES

1. Akbulut, H. and Sayman, O. (2001): An investigation on buckling of laminated plates with central square hole, *Journal of Reinforced Plastics and Composites*, 20 (13): 1112-1124.
2. Baba, B. (2007): Buckling behaviour of laminated composite plates, *Journal of Reinforced Plastics and Composites*, 26 (16): 1637-1654.
3. Baltaci, A., Sarikanat, M. and Yildiz, H. (2006): Buckling analysis of laminated composite circular plates with holes, *Journal of Reinforced Plastics and Composites*, 25 (7): 733-744.
4. Bao, G., Jiang, W. and Roberts, J. C. (1997): Analytic and finite element solutions for bending and buckling of orthotropic rectangular plates, International Journal of Solids Structures, Vol.34, No. 14, pp. 1797-1822.
5. Brown, C.J., Yettram, A.L. and Burnett, M. (1987): Stability of plates with rectangular holes, Journal of Structural Engineering, Vol. 113 No. 5, Paper No. 21466
6. Chandrashekhara, K. (1989)**:** Free vibrations of anisotropic laminated doubly curved shells, *Computers and Structures*, Vol.**33** (2), pp.435-440
7. Ghannadpour, S., Najafi, A., and Mohammadi, B. (2006): On the buckling behaviour of cross-ply laminated composite plates due to circular/elliptical cutouts. *CompositeStructures*, (75): 3-6.
8. Hyer, M. W. and Lee, H. H. (1991): The use of curvilinear fiber format to improve buckling resistance of composite plates with central circular holes, *Composite Structures*,(18): 239-261

9. Hu, H. and Lin, B. (1995): Buckling optimization of symmetrical laminated plates with various geometries and end conditions, *Composites Science and Technology*,55 (6): 277-284.
10. Ko, W. L. (1998): Anomalous Buckling Characteristics of Laminated Metal–Matrix Composite Plates with Central Square Holes, NASA TP206559, July 1998
11. Larsson, P. L. (1987): On buckling of orthotropic compressed plates with circular holes, *Composite Structures*, (7):103–121.
12. Larsson, P. L. (1989): On buckling of orthotropic stretched plates with circular holes, *Composite Structures*, (11): 121–134.
13. Lee, Y. J., Lin, H. J. and Lin, C. C. (1989): A study on the buckling behaviour of an orthotropic square plate with a central circular hole, *Composite Structures*, (13): 173–188.
14. Marshall, I. H., Little, W. and El-Tayeby, M. M. (1987): Membrane stress distributions in post-buckled composite plates with circular holes, Sixth International Conference on Composite Materials and Second European Conference on Composite Materials, Elsevier Applied Science, pp. 4.57–4.68.
15. Nemeth, M. P. (1996): Buckling and postbuckling behaviour of laminated composite plates with a cutout, NASA Technical Paper 3587, July.
16. Sahu S.K, Asha A.V, Parametric instability of twisted cross-ply laminated panels, *Aerospace Science and Technology*, vol. 15, pp. 465–475, 2011
17. Sahu, S.K., and Datta, P.K. (2003): Dynamic stability of laminated composite curved panels with cutouts, *Journal of Engineering Mechanics*, ASCE, 129(11), pp.1245-1253.
18. Saha, M., Prabhakaran, R. and Waters, W. (2004.): Compressive behaviour of pultruded composite plates with circular holes, *Composite Structures*, (65): 29-36.

19. Vandenbrink, D. J. and Kamat, M.P.(1987): Post-buckling response of isotropic and laminated composite square plates with circular hole. *Finite Elements in Analysis and Design*, (3): 165-174.
20. *Husam Al Qablan et,all.* on Assessment of the Buckling Behavior of Square Composite Plates with Circular Cutout Subjected to In-Plane Shear. *Jordan Journal of Civil Engineering,* Volume 3, No. 2, 2009
21. Walker, M. (1999): Optimal Design of Symmetric Laminates with Cutouts for Maximum Buckling Load, Composite Structures, 70 (3): 337–344.
22. Yasui, Y. (1991): The buckling of rectangular composite plate with cutout under uniaxial and biaxial compression, Proceedings of the 8th International Conference on Composite Materials, July, pp. 4-B-1–4-B-8.
23. Yoon, S. J. and Lee, W. B. (1994): Elastic buckling analysis of orthotropic plate with edge stiffener, Journal of Structural Engineering, KSCE, Vol. 14, No. 11, pp. 281-290.
24. Zhong, H., &Gu, C. (2007): Buckling of symmetrical cross-ply composite rectangular plates under a linearly varying in-plane load. Composite structures, 80(1), 42-48.